Inclusion of Space Charge
in
Longitudinal Tracking Simulations

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We start by computing the transverse electric field for a beam of uniform charge distribution in a radius $r \le a$ tranvelling in the center of a beam pipe of radius b. The transverse electric field is:

$$E_r = \begin{Bmatrix} \dfrac{\rho_0 r}{2\varepsilon_0} & r \le a \\ \dfrac{\rho_0 a^2}{2\varepsilon_0 r} & r \ge a \end{Bmatrix} \qquad [1]$$

from which one finds the scalar potential at the center of the beam:

$$\Phi = \frac{\rho_0 a^2}{2\varepsilon_0}\left(\frac{1}{2} + \ln\frac{b}{a}\right). \qquad [2]$$

We now assume that the potential is given by a similar formula when $\rho_0 = \rho_0(z)$:

$$\Phi(z) = \frac{\rho_0(z) a^2}{2\varepsilon_0}\left(\frac{1}{2} + \ln\frac{b}{a}\right). \qquad [3]$$

The beam is assumed to be moving at a uniform velocity βc, and the vector potential is related to the scalar potential

$$A_z(z) = \frac{\beta}{c}\Phi(z) \qquad [4]$$

From this one finds

$$\begin{aligned} E_z &= -\frac{\partial\Phi}{\partial z} - \frac{\partial A}{\partial t} \\ &= -\frac{\partial\Phi}{\partial z} - \frac{\beta}{c}\frac{\partial\Phi}{\partial t} \\ &= -\frac{1}{\gamma^2}\frac{\partial\Phi}{\partial z} \end{aligned} \qquad [5]$$

This is the space charge force. It acts continuously over the ring circumference, but is approximately equal to a voltage kick given once per turn at the rf cavities which is equal to

$$V(z) = E_\rho(z)C, \qquad [6]$$

where C is the circumference of the accelerator.

So the procedure is to compute $\rho(z)$ numerically based on the particle distribution. I would guess that one might need about 10 bins per rf bucket to get a reasonable idea of the space charge force. One would fit a smooth function to the binned distribution and compute the derivative. The smoothed function $\rho(z)$ can also be used to simulate feed-forward technique. The simplest method is to apply $\rho(z)$ to the cavity after a one turn delay.